① Atomic Hamiltonian

$$\underbrace{\frac{\vec{p}^2}{2m} + V(\vec{r})}_{\hat{H}_0} + \underbrace{\vec{d}\vec{E}(t)}_{\hat{H}_I}$$

② Unperturbed Hamiltonian

$|1\rangle$ Two - level atom

$\updownarrow \omega_a$

$|0\rangle$

$$\hat{I} = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\hat{H}_0 |0\rangle = \hbar\omega_0 |0\rangle$$

$$\hat{H}_0 |1\rangle = \hbar\omega_1 |1\rangle$$

$$\hat{I} H_0 \hat{I} = \hbar\omega_0 |0\rangle\langle 0| + \hbar\omega_1 |1\rangle\langle 1| =$$

$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \hbar\omega_0 & 0 \\ 0 & \hbar\omega_1 \end{pmatrix} - \frac{\hbar\omega_0}{2}\hat{I} - \frac{\hbar\omega_1}{2}\hat{I} + \ldots$$

$$= \begin{pmatrix} \frac{\hbar\omega_0}{2} - \frac{\hbar\omega_1}{2} & 0 \\ 0 & -\frac{\hbar\omega_0}{2} + \frac{\hbar\omega_1}{2} \end{pmatrix} + \hat{I} \ldots$$

$$= \frac{\hbar\omega_a}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \boxed{\frac{\hbar\omega_a}{2}\hat{\sigma}_z}$$

③ Interaction term

$$\hat{I} \cdot \vec{d}\vec{E}\hat{I}$$

$$\langle 0|\hat{d}|0\rangle = \langle 1|\hat{d}|1\rangle = 0$$

due to parity

(4) Odd parity operator
Spatial inversion

$$\vec{r} \longmapsto -\vec{r}$$ spatial inversion
$$\vec{d} \longmapsto -\vec{d}$$

$$\hat{P}|0\rangle = \pm|0\rangle$$

eigenstates atomic in a spherically symmetric potential have definite parity.

$\hat{P}$ - parity operator

$$\langle 0|\hat{d}|0\rangle =$$

$$\langle 0|P^{+}P\hat{d}P^{+}P|0\rangle =$$

$$= \langle 0|P\hat{d}P^{+}|0\rangle =$$

$= \langle 0 | -\hat{d} | 0 \rangle = 0$

*Intuition

$\langle 0 | \hat{d} | 0 \rangle = -e \langle 0 | \hat{r} | 0 \rangle$

expectation of electon position in this state

Probability is centered at zero (on nucleus), no preferred direction in space. $\langle \hat{r} \rangle = 0$

Atomic eigenstate doesn't possess

back to Hamiltonian

$$\hat{I}\,\hat{d}\vec{E}\,\hat{I} = \left(|a\rangle\langle a|\,\hat{d}\,|b\rangle\langle b| + |b\rangle\langle b|\,\hat{d}\,|a\rangle\langle a|\right)\vec{E} =$$

$$= \left(d_{ab}\,|a\rangle\langle b| + d_{ba}\,|b\rangle\langle a|\right)\vec{E}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$\hat{M} = \begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$

Depending on polarization ?

$$= \hat{\sigma}_x \cdot \mathcal{E}_0 \cos\omega t \cdot |d| =$$

$$= \hbar\Omega\cos(\omega t)\,\hat{\sigma}_x$$

Total Hamiltonian

$$\frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\Omega\cos(\omega t)\hat{\sigma}_x$$

5. Rotating-wave Appx-n

$$\hat{H}_{RWA} = UHU^{+} + iU^{+}\frac{dU}{dt} \quad ?$$

transformation

$$\hat{u} = e^{i\omega t/2\,\hat{\sigma}_z}$$

BCH formula.

$$H_{rF} = \frac{\delta}{2}\hat{\sigma}_z + \frac{\Omega}{2}\cos\varphi\,\hat{\sigma}_x + \frac{\Omega}{2}\sin\varphi\,\hat{\sigma}_y$$

Should I keep the derivation
is sign correct?