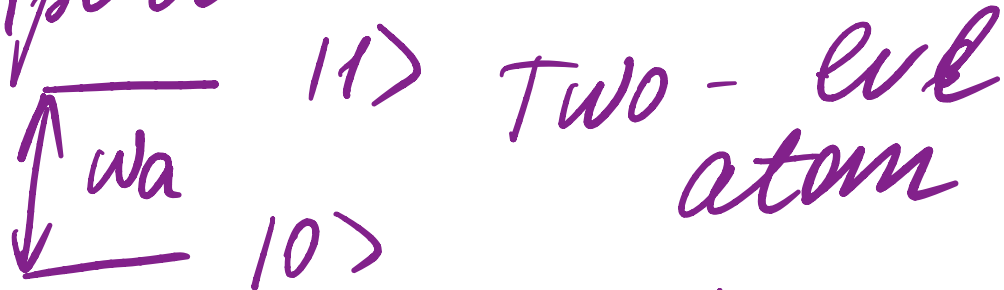


# ① Atomic Hamiltonian

$$\underbrace{\frac{\vec{p}^2}{2m} + V(\vec{r})}_{\hat{H}_0} + \underbrace{\vec{d} \cdot \vec{E}(t)}_{\hat{H}_I}$$

# ② Unperturbed Hamiltonian



$$\hat{I} = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\hat{H}_0 |0\rangle = \hbar\omega_0 |0\rangle$$

$$\hat{H}_0 |1\rangle = \hbar\omega_1 |1\rangle$$

$$\hat{I} \hat{H}_0 \hat{I} = \hbar\omega_0 |0\rangle\langle 0| + \hbar\omega_1 |1\rangle\langle 1|$$

$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \hbar\omega_0 & 0 \\ 0 & \hbar\omega_1 \end{pmatrix} - \frac{\hbar\omega_0}{2} \hat{I} - \frac{\hbar\omega_1}{2} \hat{I} + \dots$$

$$= \begin{pmatrix} \frac{\hbar\omega_0}{2} - \frac{\hbar\omega_1}{2} & 0 \\ 0 & -\frac{\hbar\omega_0}{2} + \frac{\hbar\omega_1}{2} \end{pmatrix} + \hat{I} \dots$$

$$= \frac{\hbar\omega_a}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar\omega_a}{2} \hat{\sigma}_z$$

③ Interaction term

$$\hat{I} \cdot \vec{d} \vec{E} \hat{I}$$

$$\langle 0 | \hat{d} | 0 \rangle = \langle 1 | \hat{d} | 1 \rangle = 0$$

due to parity

① Odd parity operator  
Spatial inversion

$$\begin{aligned}\vec{r} &\mapsto -\vec{r} \\ \vec{d} &\mapsto -\vec{d}\end{aligned} \quad \text{spatial inversion}$$

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$$\hat{P} |0\rangle = \pm |0\rangle$$

eigenstates of an atom in a spherically symmetric potential have definite parity.

$\hat{P}$  - parity operator

$$\langle 0 | \hat{d} | 0 \rangle =$$

$$\langle 0 | P^\dagger P \hat{d} P^\dagger P | 0 \rangle =$$

$$= \langle 0 | P \hat{d} P^\dagger | 0 \rangle =$$

$$= \langle 0 | -\hat{d} | 0 \rangle = 0$$

\* Intuition

$$\langle 0 | \hat{d} | 0 \rangle = -e \langle 0 | \hat{r} | 0 \rangle$$

expectation of electron position in this state

Probability is centered at zero (on nucleus), no preferred direction in space.  $\langle \hat{r} \rangle = 0$

Atomic eigenstate doesn't possess

Back to Hamiltonian

$$\hat{I} \hat{d} \vec{E} \hat{I} = (|a \times a\rangle \hat{d} |b \times b\rangle + |b \times b\rangle \hat{d} |a \times a\rangle) \vec{E} =$$

$$\hat{M} (d_{ab} |a \times b\rangle + d_{ba} |b \times a\rangle) \vec{E}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |0 \times 1\rangle + |1 \times 0\rangle$$

$$\hat{M} = \begin{pmatrix} |0 \times 0\rangle & |0 \times 1\rangle \\ |1 \times 0\rangle & |1 \times 1\rangle \end{pmatrix}$$

Depending on polarization (?)

$$\hat{\sigma}_x \cdot \hat{E}_0 \cos \omega t \cdot |d| =$$

$$= \hbar \Omega \cos(\omega t) \hat{\sigma}_x$$

# Total Hamiltonian

$$\left| \frac{\hbar \omega a}{2} \hat{\sigma}_z + \hbar \Omega \cos(\omega t) \hat{\sigma}_x \right.$$

5. Rotating-wave *Approx - n*

$$\hat{H}_{RWA} = U H U^\dagger + i U^\dagger \frac{dU}{dt} \quad (?)$$

transformation

$$\hat{U} = e^{i\omega t / 2 \hat{\sigma}_z}$$

BCH formula.

$$H_{RF} = \frac{\Omega}{2} \hat{\sigma}_z + \frac{\Omega}{2} \cos \varphi \hat{\sigma}_x +$$

$$\oplus \frac{\Omega}{2} \sin \varphi \hat{\sigma}_y$$

Should I keep the derivation is sign correct?