Quantum computation and simulation with neutral atoms

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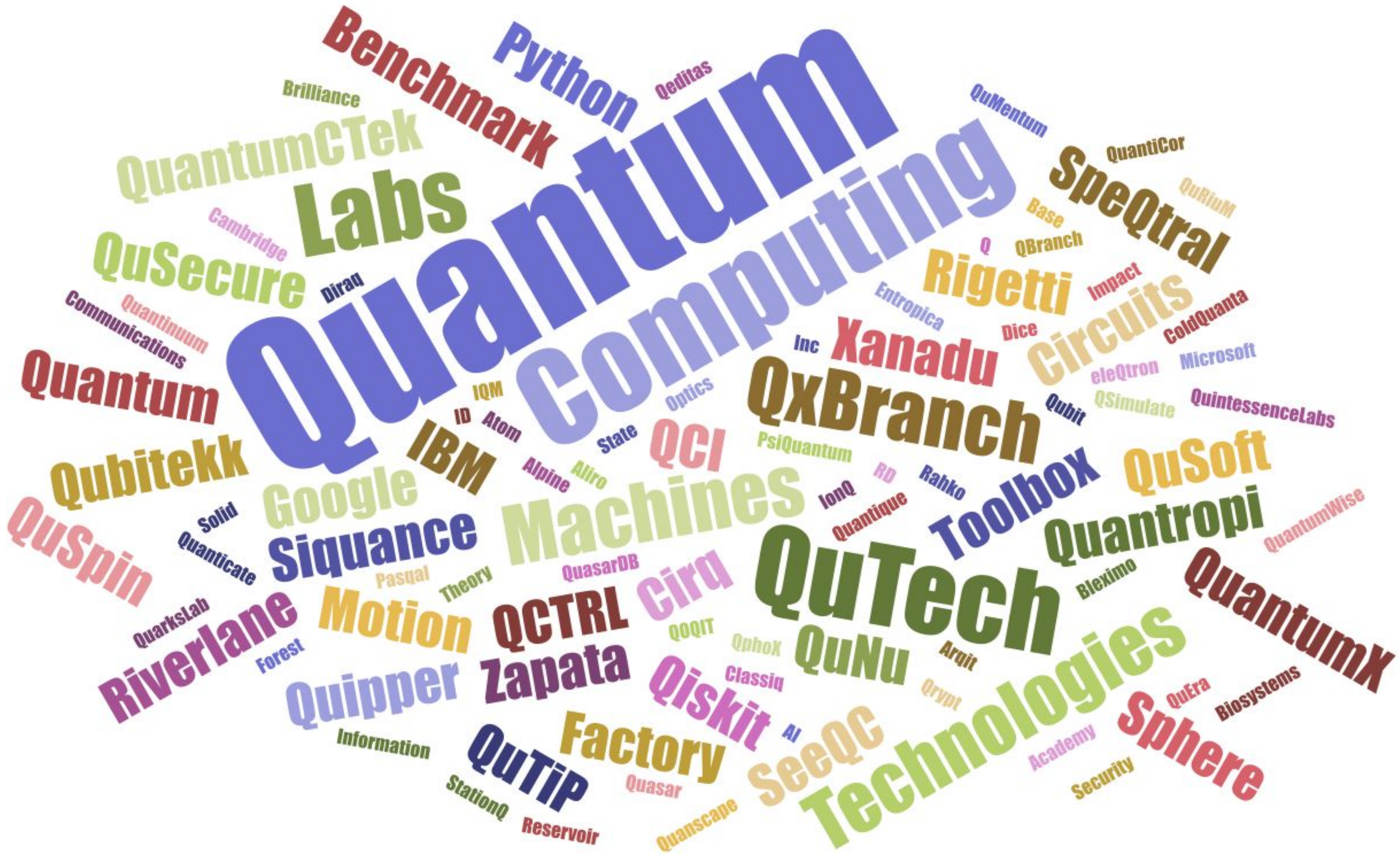
Quantum computing



Which system requires a quantum simulator?

- A quantum system
- Cannot be measured
- The laws describing are unknown
- Large number of equations

Illustration by Kharlamova

Agenda

1. Basics of quantum computation
   - define qubits and basic operations
   - neutral atom single-qubit gates
2. Experiment: universal single-qutrit gate
   - how to expand the space and why?
   - specific realization and results
3. Experiment: holonomic quantum computation
   - an alternative quantum computation approach
   - experimental realization and results
4. Quantum computing for lattice gauge theory
   - Kogut-Susskind Hamiltonian
   - quantum computing algorithm

Part 1

1. Basics of quantum computation
   - define qubits and basic operations
   - neutral atom single-qubit gates
2. Universal qutrit gate
   - how to expand the space and why?
   - specific realization and results
3. Holonomic quantum computation
   - an alternative quantum computation approach
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   - Kogut-Susskind Hamiltonian
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Qubit

- Classical bits has two available states On/Off
- Quantum bits (qubits) could be in the superposition states

$$|\psi\rangle = \alpha|0\rangle + e^{i\phi}\beta|1\rangle$$

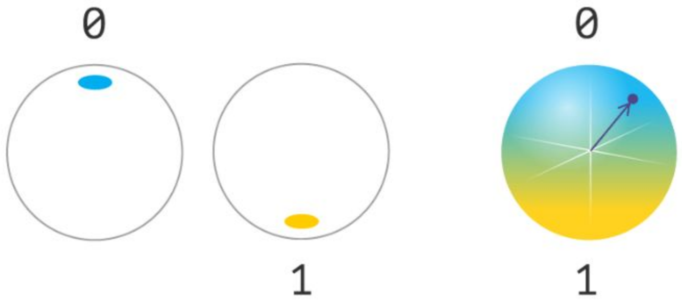
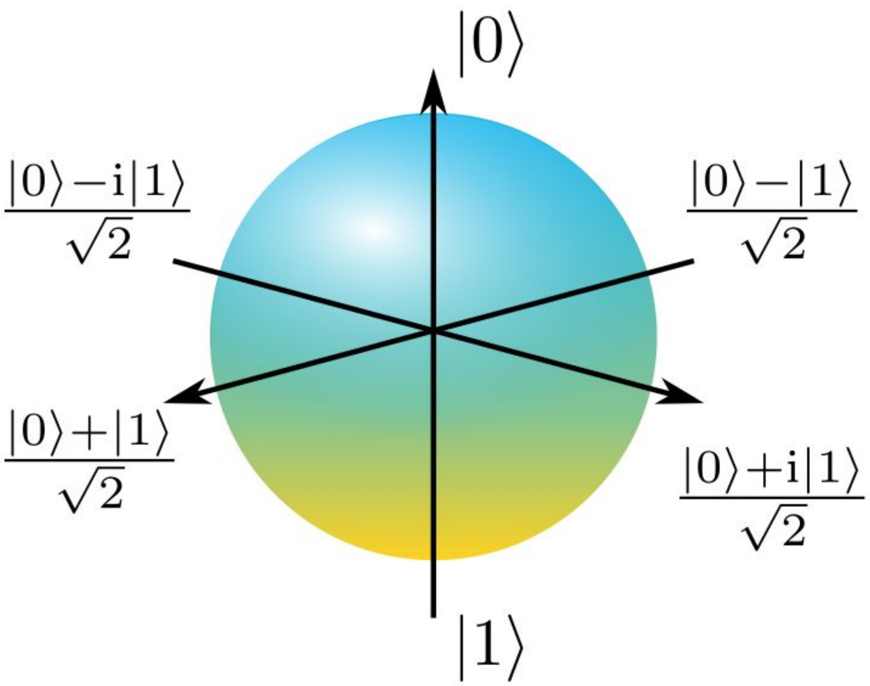


Illustration from https://1qbit.com/

The Bloch sphere

- Unit sphere
- Main axis of a Bloch sphere are eigenvectors of Pauli matrices

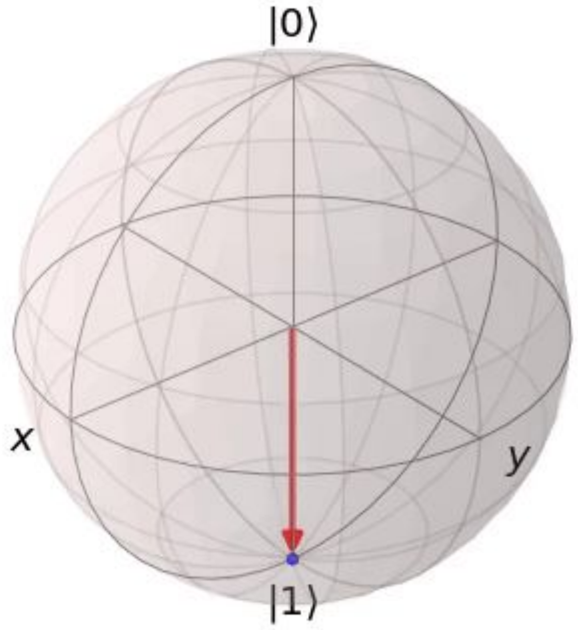


Manipulating the qubit

- We need a unitary operator to induce rotation of our qubit
- Pauli matrices are generators of unitary rotations on a sphere!

- For example, rotation happens around *y* axis by '*t*' angle

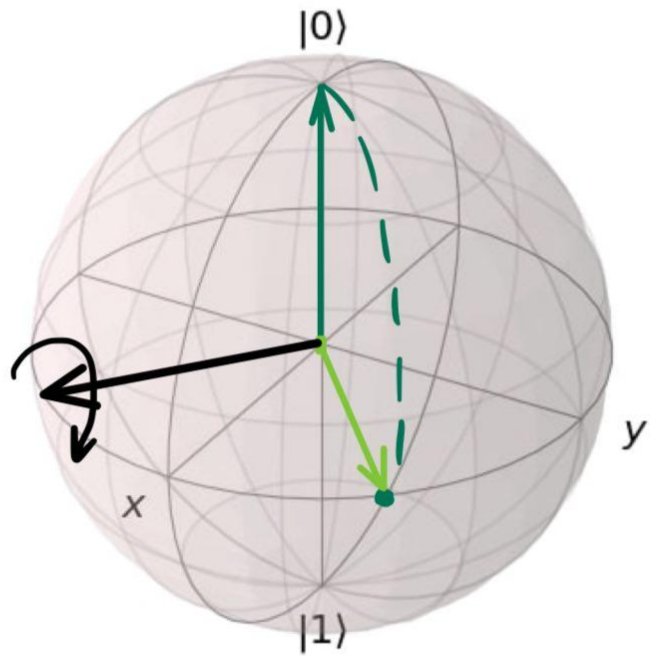
$$e^{-\mathrm{i}\hat{\sigma}_y t/2} = \begin{pmatrix} \cos(t/2) & -\sin(t/2) \\ \sin(t/2) & \cos(t/2) \end{pmatrix}$$



Arbitrary rotation around arbitrary axis

- We can find an arbitrary axis around which we need to rotate

$$e^{-\mathrm{i}t\vec{n}\cdot\vec{\sigma}/2} = \cos\left(\frac{t}{2}\right)\hat{I} - \mathrm{i}\sin\left(\frac{t}{2}\right)\vec{n}\cdot\vec{\sigma}$$



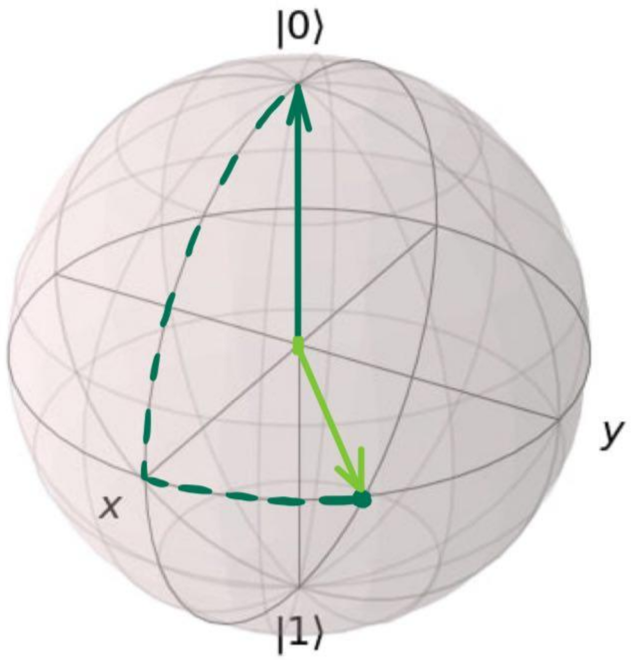
Arbitrary rotation through decomposition

- We can find an arbitrary axis around which we need to rotate

$$e^{-\mathrm{i}t\vec{n}\cdot\vec{\sigma}/2} = \cos\left(\frac{t}{2}\right)\hat{I} - \mathrm{i}\sin\left(\frac{t}{2}\right)\vec{n}\cdot\vec{\sigma}$$

- Any unitary transformation could be decomposed from these generic rotation as an example

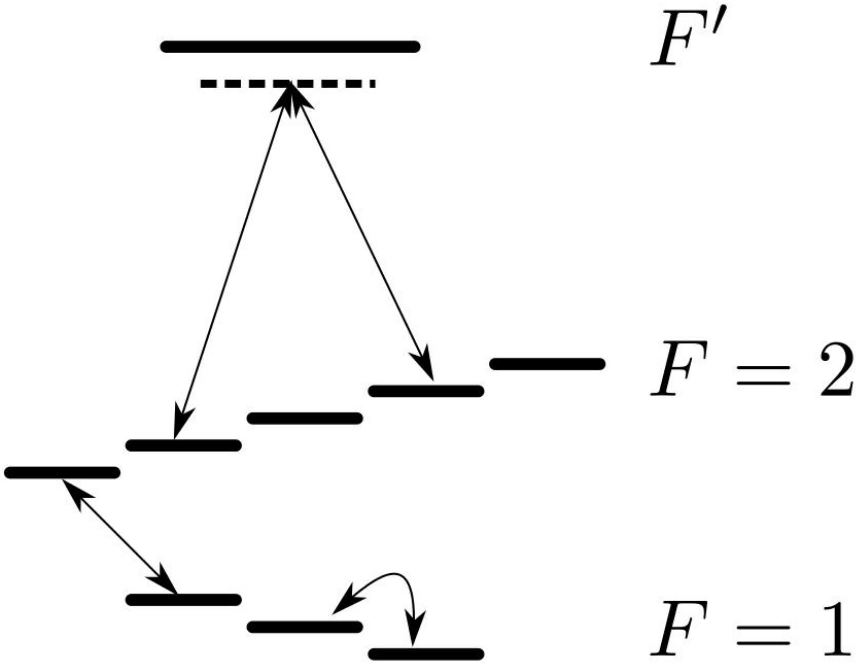
$$\hat{U} = e^{\mathrm{i}\alpha}R_z(\beta)R_y(\gamma)R_z(\delta)$$



In review

- We know what is a qubit and how to rotate it if we have Pauli matrices handy.

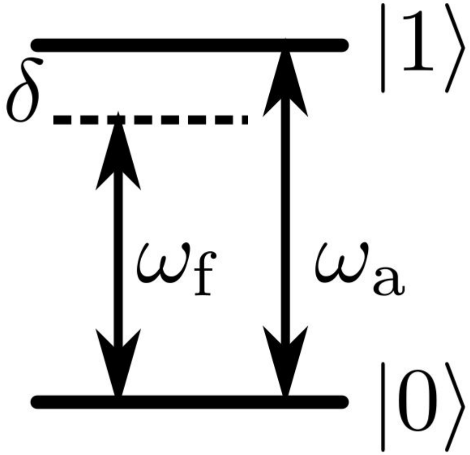
- Choose magnetic sublevels
- Couple them the way you want
  - within a single *F* manifold
  - through a microwave field coupling different *F*
  - optically coupling through a third level



Physical realization: driven two-level atom

- Hamiltonian

$$\hat{H} = \frac{\omega_{\mathrm{a}}}{2}\sigma_z + \Omega \cdot \sigma_x \cos(\omega_{\mathrm{f}} t + \phi)$$



- Evolution is described by superposition of Pauli matrices

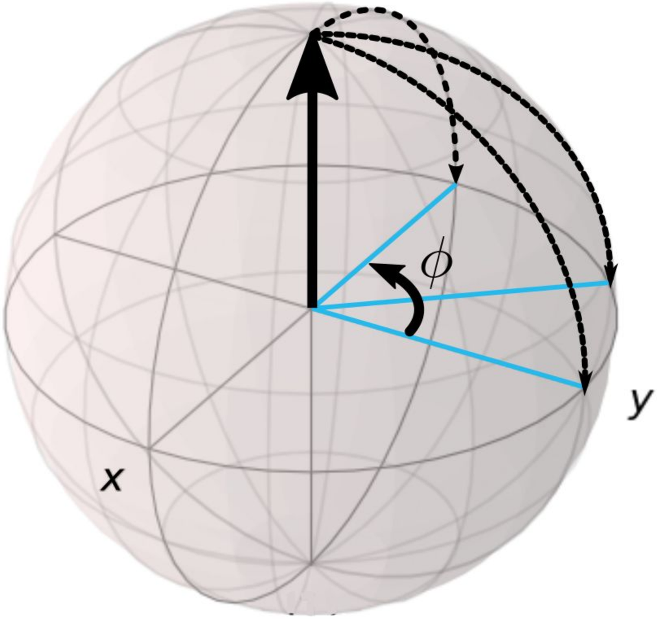
$$\hat{U} = \hat{I}\cos(\chi) + \frac{\mathrm{i}\sin(\chi)}{\Omega_{\mathrm{eff}}}\left(\frac{\delta}{2}\sigma_z + \frac{|\Omega|}{2}(\sigma_x\cos\phi - \sigma_y\sin\phi)\right)$$

$\Omega_{\mathrm{eff}} = \sqrt{\frac{\delta^2}{4} + \frac{|\Omega|^2}{4}}$ and $\chi = t\Omega_{\mathrm{eff}}$

Field defines a gate

- Rabi frequency dictates how fast qubit rotates
- Phase of the field selects along which longitude qubit rotates

$$\hat{U} = \begin{pmatrix} \cos(\Omega t/2) & \mathrm{i}e^{\mathrm{i}\phi}\sin(\Omega t/2) \\ \mathrm{i}e^{-\mathrm{i}\phi}\sin(\Omega t/2) & \cos(\Omega t/2) \end{pmatrix}$$



In review

- how to prepare and manipulate a qubit in a two-level atom

Questions?

Part 2

1. Basics of quantum computation
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What is a qudit?

Qudit is a superposition of d levels

$$|\alpha\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \cdots + \alpha_{d-1}|d-1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{d-1} \end{pmatrix} \in \mathbb{C}^{\mathrm{d}}$$

Qudit benefits

- A larger state space to store and process information
- An ability to do multiple control operations simultaneously
- Reduction of the circuit complexity

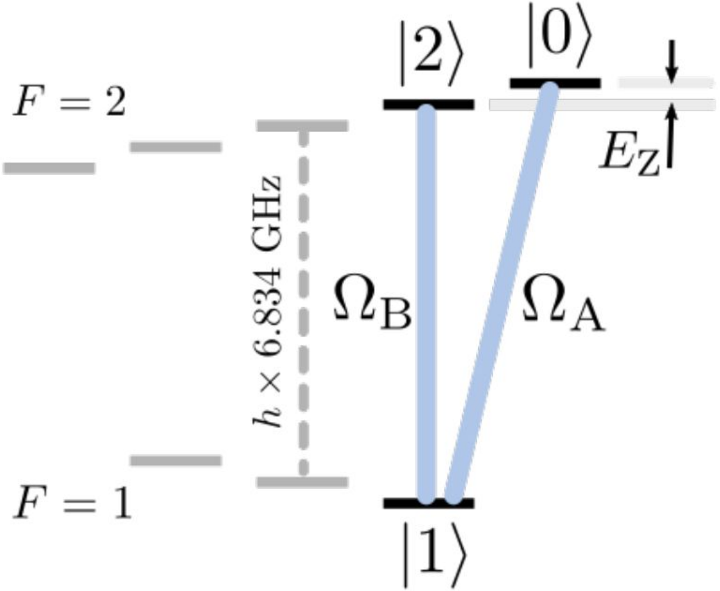
Wang, Yuchen, et al. Frontiers in Physics 8 (2020): 589504.

Qutrit and SU(3)

- Qutrit is more quantum then qubit
- Described by eight Gell-Mann matrices
- Strong interactions are described by the same algebra
- Can we relate it to physics which happens inside of nucleus?

Majorana Stellar representation of a qutrit
https://heyredhat.github.io

- We choose three levels
- Two couplings are freely available through microwave transitions
- How to create the third coupling between $|0\rangle$ and $|2\rangle$?



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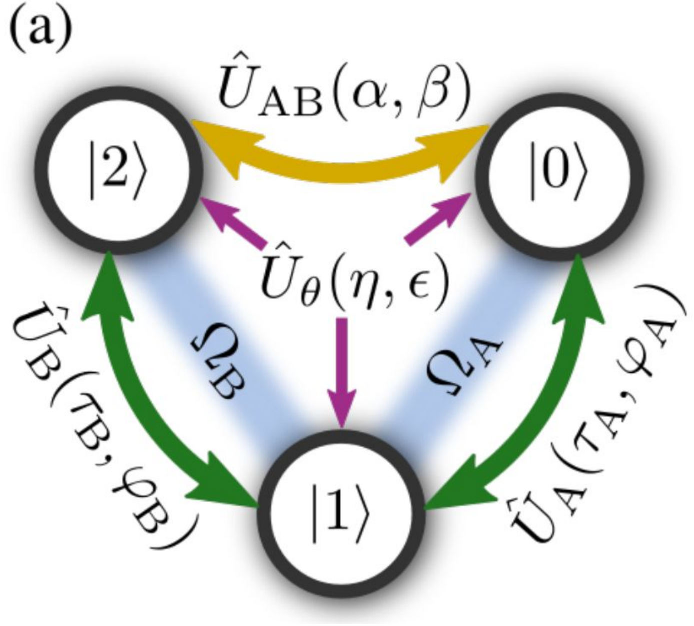
We can represent it differently

- through SU(2)

$$\hat{U}^{\mathrm{I}}_{\mathrm{gen.}} = \hat{U}_{\theta}(\eta, \epsilon)\hat{U}_{\mathrm{B}}(\tau_{\mathrm{B2}}, \varphi_{\mathrm{B2}})\hat{U}_{\mathrm{A}}(\tau_{\mathrm{A1}}, \varphi_{\mathrm{A1}})\hat{U}_{\mathrm{B}}(\tau_{\mathrm{B1}}, \varphi_{\mathrm{B1}}),$$

- through a simultaneous coupling

$$\hat{U}^{\mathrm{II}}_{\mathrm{gen.}} = \hat{U}_{\theta}(\eta, \epsilon)\hat{U}_{\mathrm{B}}(\tau_{\mathrm{B2}}, \varphi_{\mathrm{B2}})\hat{U}_{\mathrm{A}}(\tau_{\mathrm{A1}}, \varphi_{\mathrm{A1}})\hat{U}_{\mathrm{AB}}(\alpha, \beta)$$



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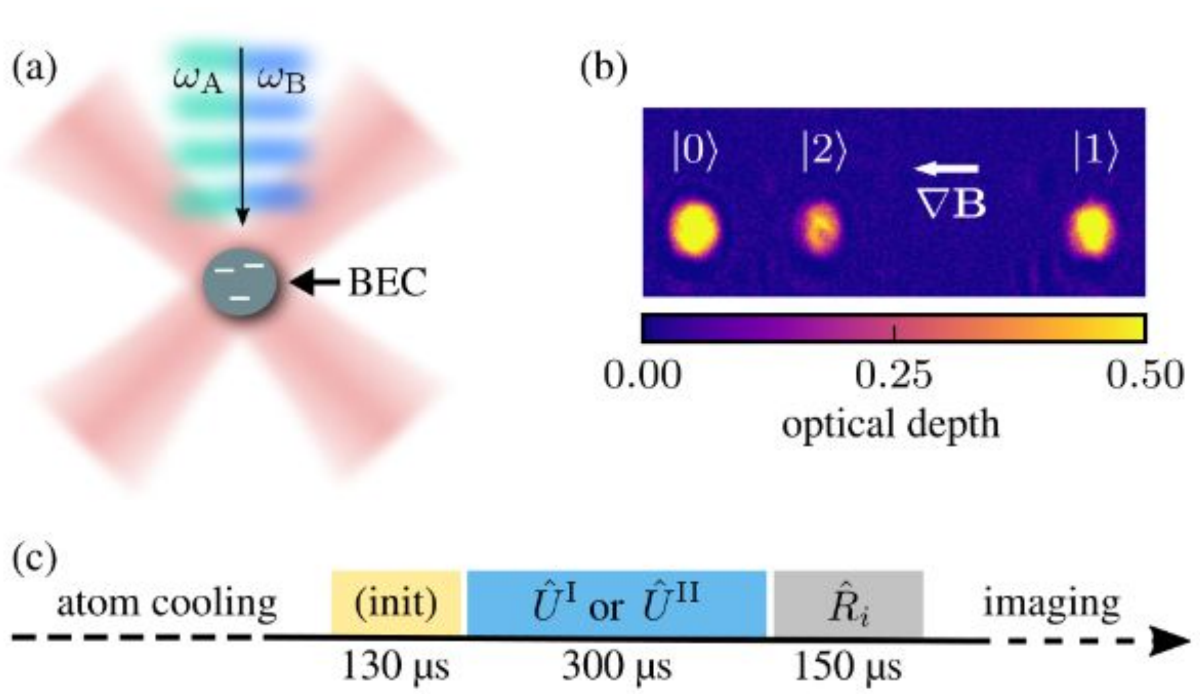
Experimental sequence



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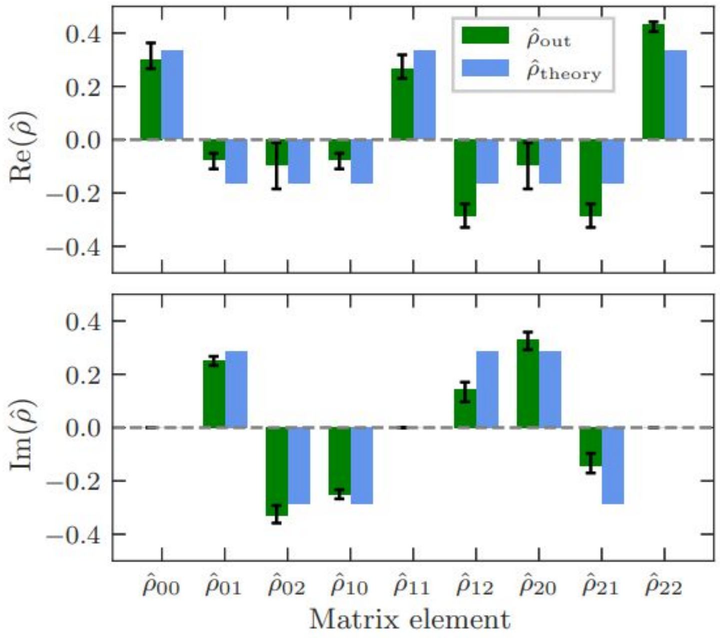
- We prepare a Walsch-Hadamard gate
- We use eight RF pulses for the state tomography
- We reconstruct the full density matrix via maximum likelihood
- We find fidelity to be 90 – 95%



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- How to create a universal set of single-qutrit gates

Questions?

Part 3

1. Basics of quantum computation
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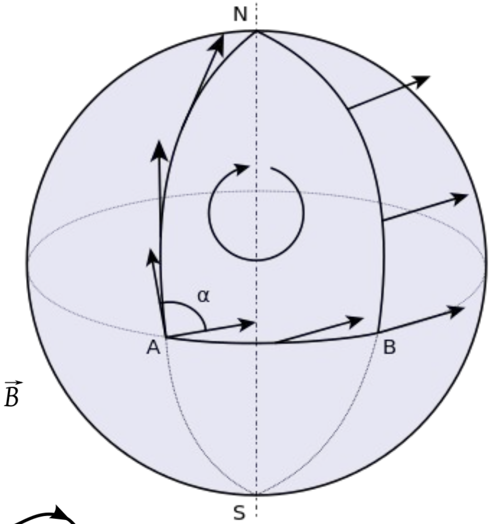
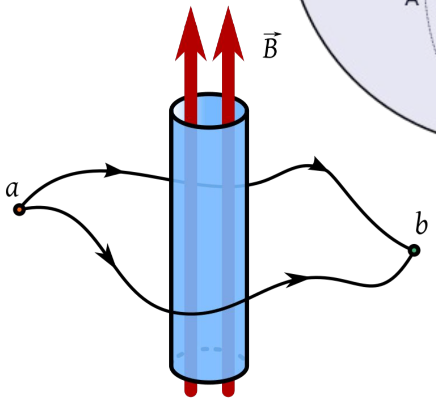
- Vector travelling in a curved space obtains a phase even after returning to an initial state
- Classic but quantum example: Aharonov-Bohm effect

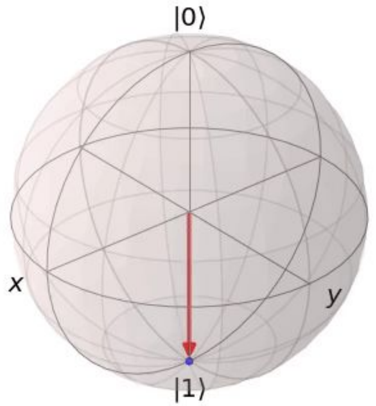




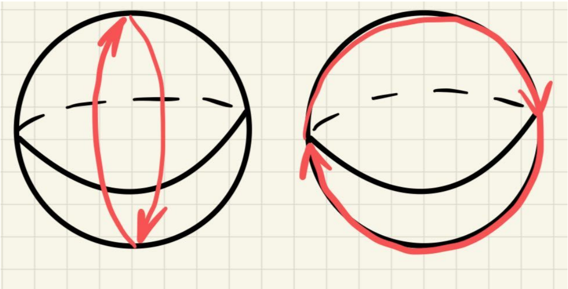
Wikipedia

Dynamical vs geometric gates

- Before we were switching on Hamiltonian for some time
- Now we vary Hamiltonian in time adiabatically, but we start and end in the same state

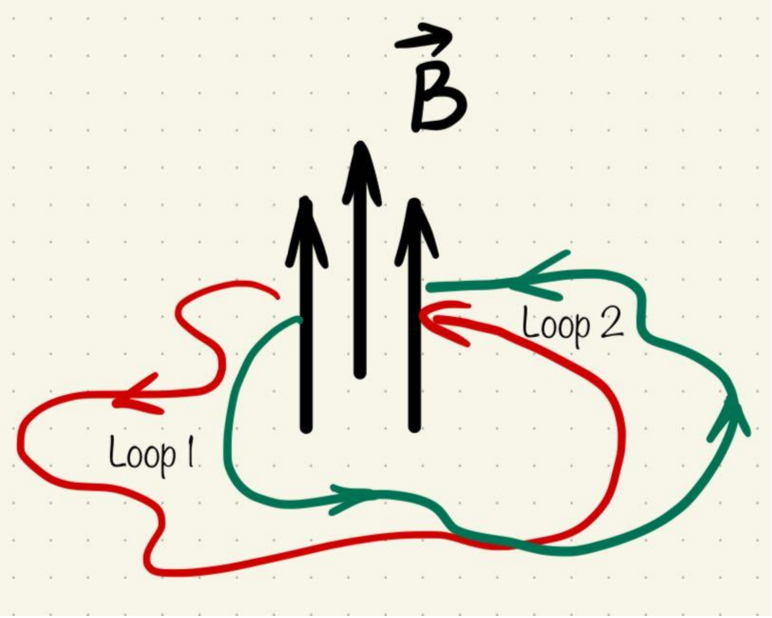


dynamical gate



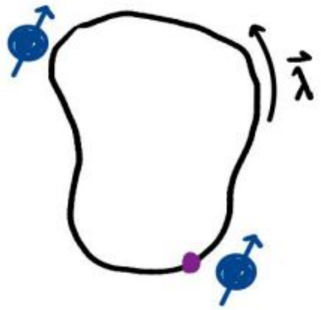
geometric gate

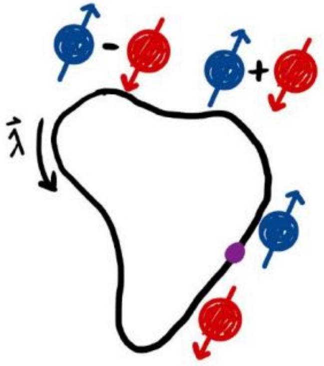
- Loop deformations would not change a gate if the loop contains the same flux
- Noise resilience



Universality

- Abelian phase: corresponds to just getting a phase factor - geometric phase
- Non-abelian phase: if we have multiple degenerate levels, we can land into a superposition of multiple levels.



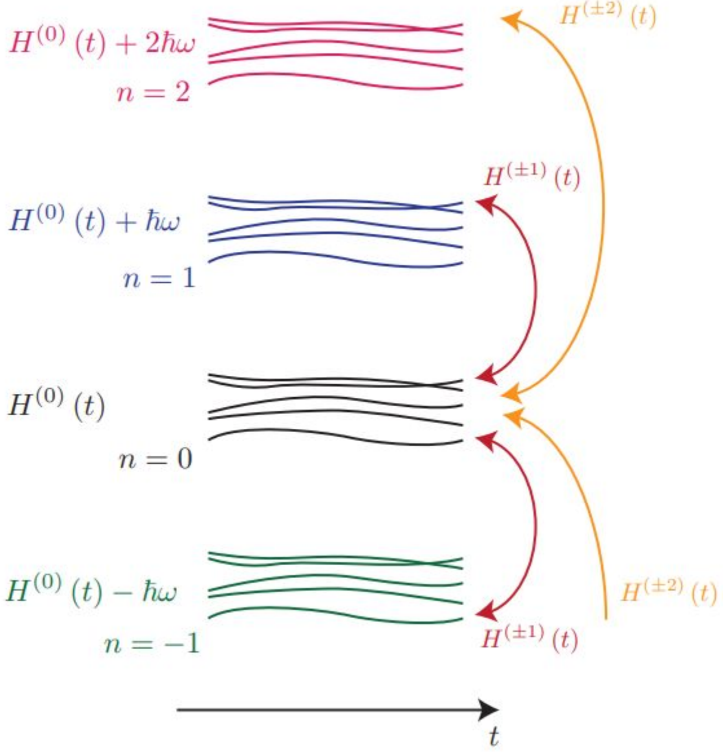


- Let's modulate Hamiltonian at a fast frequency

$$\hat{H} = \Omega_0 \underbrace{\mathbf{q}(t)}_{\text{slow changing parameters}} \cdot \hat{\mathbf{F}} \cos \omega t$$

slow changing parameters

- If we look into the 0th Floquet band, we have a handful of degenerate levels



Novičenko V, Juzeliūnas G. PRA. 2019; 100(1):012127.

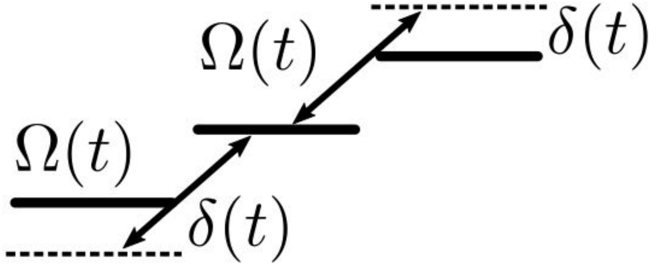
In review

- Geometric phase
- Holonomic gates with abelian and non-abelian phases
- Floquet engineering to create degeneracies

- Spin in a magnetic field with fast modulation

$$\hat{H} = \Omega_0 \, \mathbf{q}(t) \cdot \hat{\mathbf{F}} \cos \omega t$$

- Periodically modulate the phase, amplitude, and central frequency of the driving field



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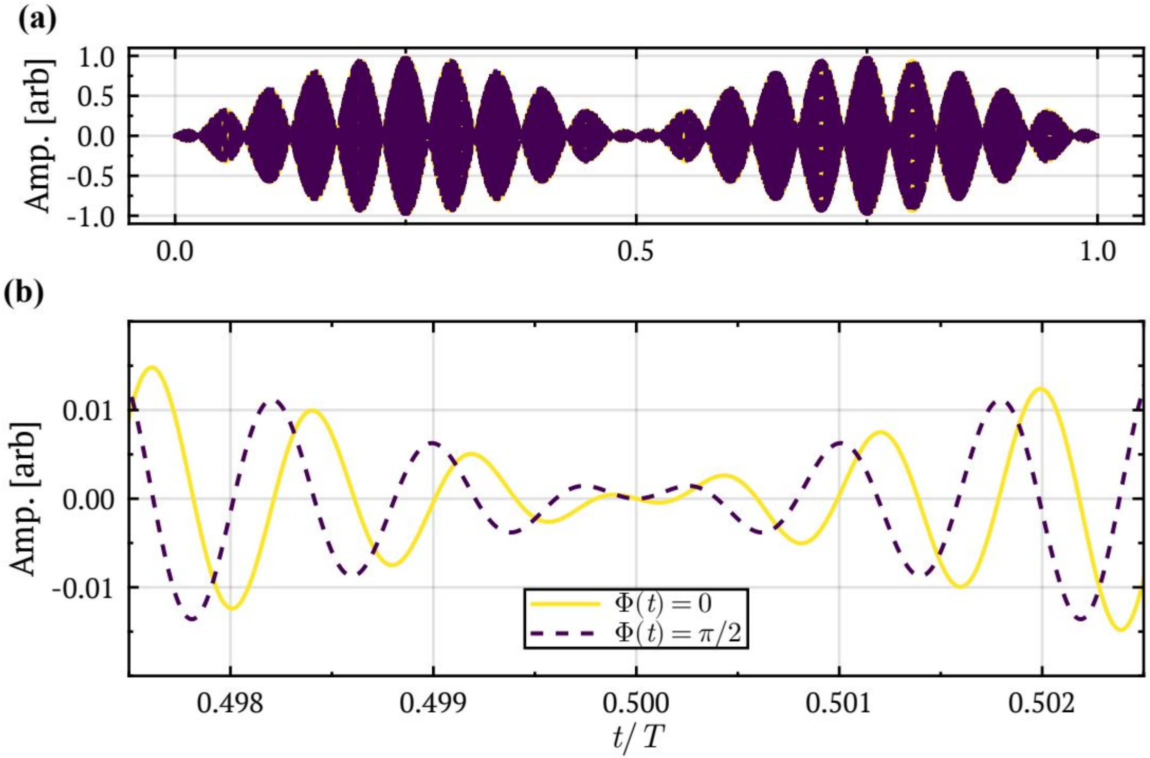
Example of modulation



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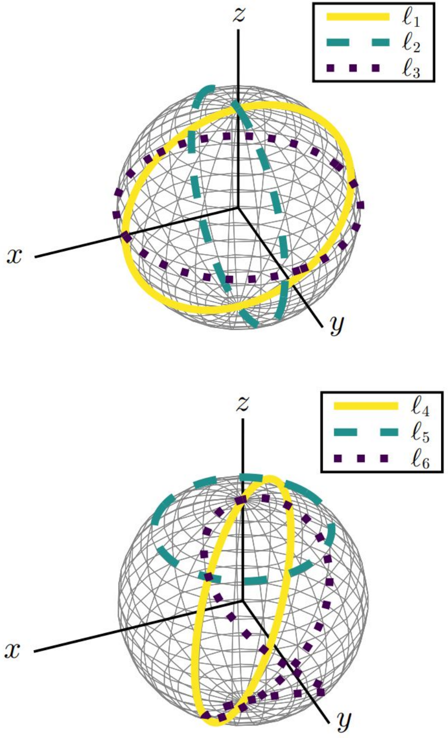
Different loops results in different evolution operators, examples

Loops	$\Theta(t)$	$\Phi(t)$	$\hat{\Gamma}_A(\ell)$
ℓ_1	Ωt	0	$\exp\left(-i2\pi g\hat{F}_y/\hbar\right)$
ℓ_2	Ωt	$\pi/2$	$\exp\left(i2\pi g\hat{F}_x/\hbar\right)$
ℓ_3	$\pi/2$	Ωt	$\exp\left(-i2\pi g\hat{F}_z/\hbar\right)$
ℓ_4	Ωt	$\pi/4$	$\exp\left[i\sqrt{2}\pi g\left(\hat{F}_x-\hat{F}_y\right)/\hbar\right]$
ℓ_5	$\pi/4$	Ωt	-
ℓ_6	Ωt	Ωt	-



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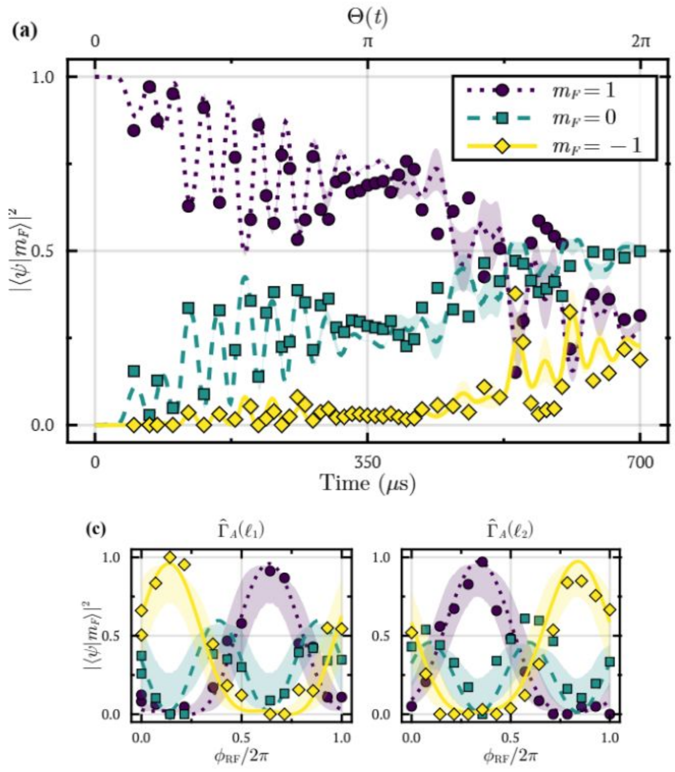
Our experimental results

- We prepare cold atomic cloud and apply modulated field
- Tomography of the final state
- We obtain different gates at the end of the loop



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In review

- How to create a universal set of single-qubit holonomic set

Questions?

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Connection to quantum simulation

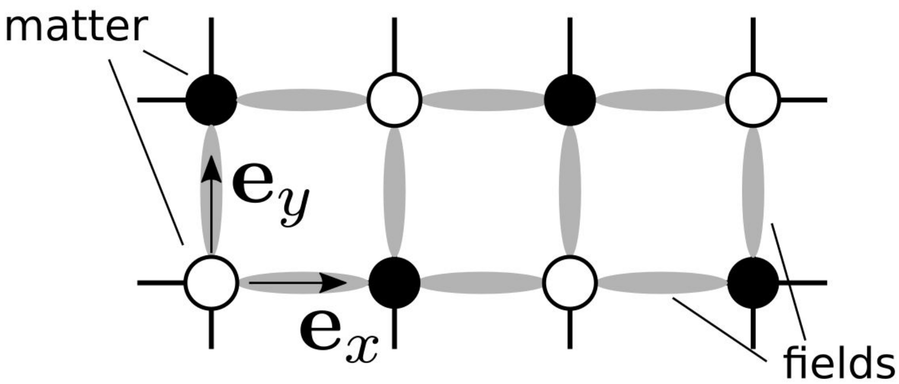
- Lattice-gauge theories
- Artificial fields
- Dynamical fields

- Meth, Michael, et al. "Simulating 2D lattice gauge theories on a qudit quantum computer." *arXiv preprint arXiv:2310.12110* (2023).
- González-Cuadra, Daniel, et al. "Hardware efficient quantum simulation of non-abelian gauge theories with qudits on Rydberg platforms." *Physical Review Letters* 129.16 (2022): 160501.
- Ohler, Simon, et al. "Self-generated quantum gauge fields in arrays of Rydberg atoms." *New Journal of Physics* 24.2 (2022): 023017.

Illustration by Kharlamova

Lattice-gauge theory basics

- discretization of space and time
- representing matter and fields on a two-dimensional lattice
- staggered matter description



$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B + m\hat{H}_m + \Omega \hat{H}_k,$$

- even lattice sites- particles (black)
- odd lattice sites - anti-particles (white)

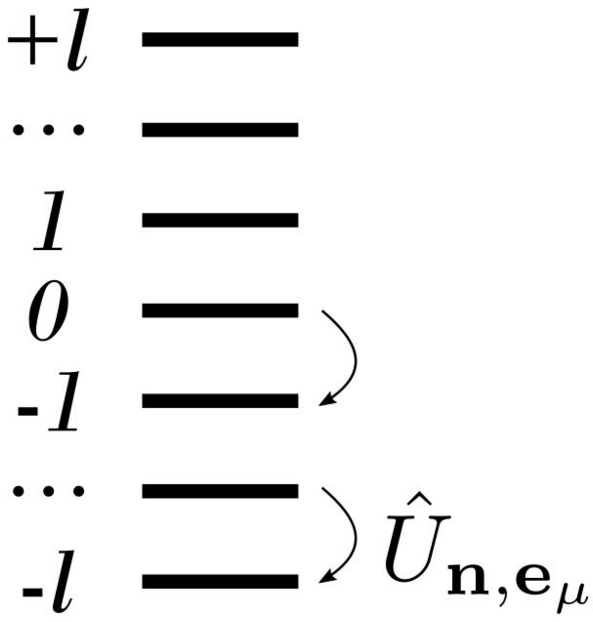
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

Gauge field description

- bosonic field has infinite but discrete spectrum
- to describe electric field we need electric field operator and lowering operator

$$\hat{E}_{\mathbf{n},\mathbf{e}_\mu} \left|E_{\mathbf{n},\mathbf{e}_\mu}\right\rangle = E_{\mathbf{n},\mathbf{e}_\mu} \left|E_{\mathbf{n},\mathbf{e}_\mu}\right\rangle$$

$$\hat{U}_{\mathbf{n},\mathbf{e}_\mu} \left|E_{\mathbf{n},\mathbf{e}_\mu}\right\rangle = \left|E_{\mathbf{n},\mathbf{e}_\mu} - 1\right\rangle$$



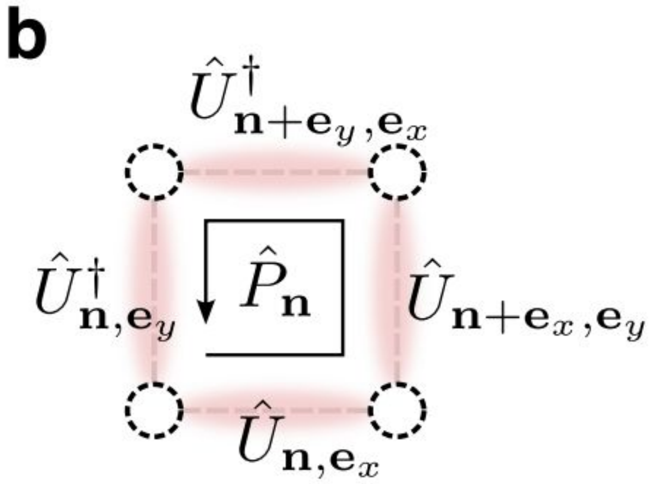
- magnetic energy is calculated through a loop over a plaquette loop

$$\hat{P}_{\mathbf{n}} = \hat{U}_{\mathbf{n},\mathbf{e}_x} \hat{U}_{\mathbf{n}+\mathbf{e}_x,\mathbf{e}_y} \hat{U}^{\dagger}_{\mathbf{n}+\mathbf{e}_y,\mathbf{e}_x} \hat{U}^{\dagger}_{\mathbf{n},\mathbf{e}_y} .$$

- energy contained in gauge fields

$$\hat{H}_E = \frac{1}{2} \sum_{\mathbf{n}} \left(\hat{E}^2_{\mathbf{n},\mathbf{e}_x} + \hat{E}^2_{\mathbf{n},\mathbf{e}_y} \right) ,$$

$$\hat{H}_B = -\frac{1}{2} \sum_{\mathbf{n}} \left(\hat{P}_{\mathbf{n}} + \hat{P}^{\dagger}_{\mathbf{n}} \right) ,$$



Mass term

- fermionic matter operators

$$\hat{H}_m = \sum_{\mathbf{n}} (-1)^{n_x + n_y} \hat{\phi}^\dagger_{\mathbf{n}} \hat{\phi}_{\mathbf{n}},$$

Kinetic term

- kinetic term which corresponds to interaction between field and matter
- responsible for particle-antiparticle creation

$$\hat{H}_k = \sum_{\mathbf{n}} \sum_{\mu=x,y} \left(\hat{\phi}_{\mathbf{n}} \hat{U}^{\dagger}_{\mathbf{n},\mathbf{e}_\mu} \hat{\phi}^{\dagger}_{\mathbf{n}+\mathbf{e}_\mu} + \mathrm{H.c.} \right)$$

We have our Hamiltonian which we like!

$$\hat{H} = \boxed{g^2 \hat{H}_E} + \boxed{\frac{1}{g^2} \hat{H}_B} + \boxed{m \hat{H}_m} + \boxed{\Omega \hat{H}_k},$$

electric

magnetic

mass

kinetic terms

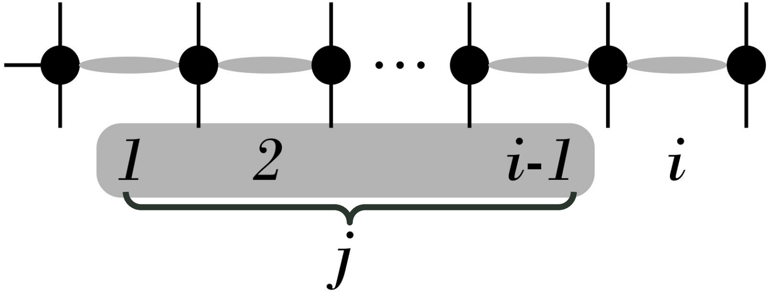
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

Matter subsystem in terms of ‘qubits’

- Jordan-Wigner transformation for matter

$$\hat{\phi}_i = \prod_{j<i}(-\hat{\sigma}_j^z)\hat{\sigma}_i^-, \quad \hat{\phi}_i^\dagger = \prod_{j<i}(-\hat{\sigma}_j^z)\hat{\sigma}_i^+$$

- additional phase is introduced to ensure fermionic anti-commutation relations
- this phase is nonlocal, all modes prior (j<i) play a role



The phase is determined by the number of occupied fermionic modes in modes. The phase is +1 if the number of occupied modes is even
−1 if the number of occupied modes is odd.

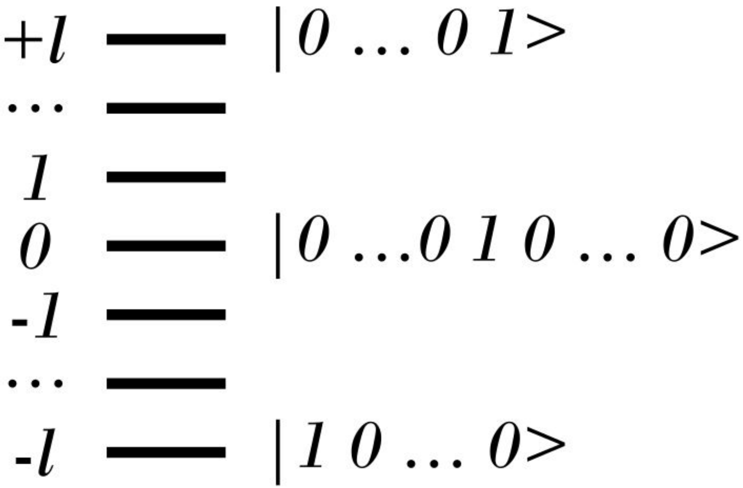
P. Jordan and E. P. Wigner, Über das paulische äquivalenzverbot, Z. Phys. 47, 631 (1928)

Gauge fields in terms of 'quDits'

- truncated ladder
- qudits are natural solutions (*)
- qubits can be used as well
- two operators that we care about

$$\hat{E} \mapsto \hat{S}^z = \frac{1}{2} \sum_{i=1}^{2l} \prod_{j=1}^{i} \hat{\sigma}_j^z,$$

$$\hat{U} \mapsto \sum_{i=1}^{2l} \hat{\sigma}_i^- \hat{\sigma}_{i+1}^+,$$

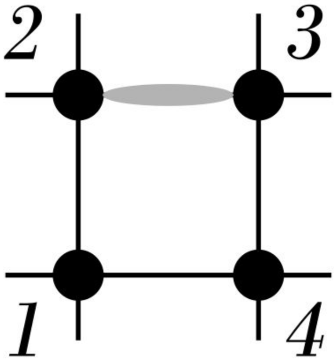


$$|-l+j\rangle = |\overbrace{0\ldots0}^{j}\,1\,\overbrace{0\ldots0}^{2l-j}\rangle,$$

(*) Meth, Michael, et al. arXiv preprint arXiv:2310.12110 (2023).
Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

Hamiltonian in terms of Pauli matrices

- 1–4 fermionic modes of a single plaquette
- 5–7 bosonic modes



$$\hat{H}_E = \frac{g^2}{4}\Big\{\hat{\sigma}_5^z\left[\hat{\sigma}_1^z - \hat{\sigma}_3^z + \hat{\sigma}_6^z(\hat{\sigma}_1^z - \hat{\sigma}_3^z - 2) - 1\right] + \hat{\sigma}_2^z\left[\hat{\sigma}_1^z + 2\hat{\sigma}_5^z\left(\hat{\sigma}_6^z + 1\right) - 1\right] + 4\hat{\sigma}_6^z\Big\},$$

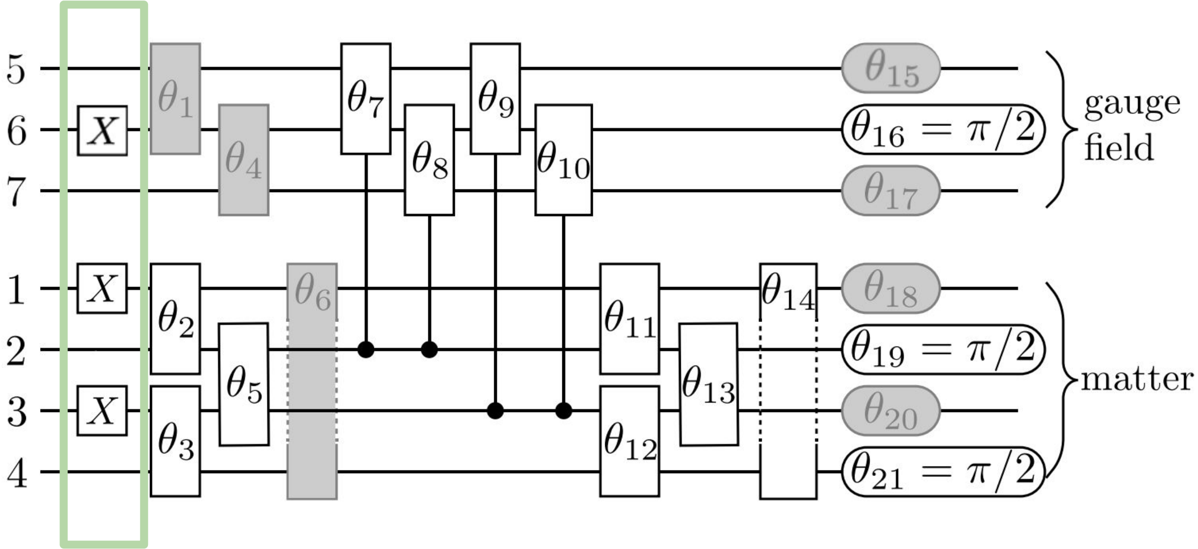
$$\hat{H}_B = -\frac{1}{2g^2}\left[\hat{\sigma}_6^+\left(\hat{\sigma}_5^- + \hat{\sigma}_7^-\right) + \hat{\sigma}_6^-\left(\hat{\sigma}_5^+ + \hat{\sigma}_7^+\right)\right],$$

$$\hat{H}_m = \frac{m}{2}\left(\hat{\sigma}_1^z - \hat{\sigma}_2^z + \hat{\sigma}_3^z - \hat{\sigma}_4^z\right),$$

$$\hat{H}_{\text{kin}} = -i\Omega\Big[\hat{\sigma}_1^+\hat{\sigma}_2^- + \hat{\sigma}_1^+\hat{\sigma}_4^- + \hat{\sigma}_4^-\hat{\sigma}_3^+ - \hat{\sigma}_2^-\left(\hat{\sigma}_5^+\hat{\sigma}_6^- + \hat{\sigma}_6^+\hat{\sigma}_7^-\right)\hat{\sigma}_3^+\Big] + \text{H.c.}$$

Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

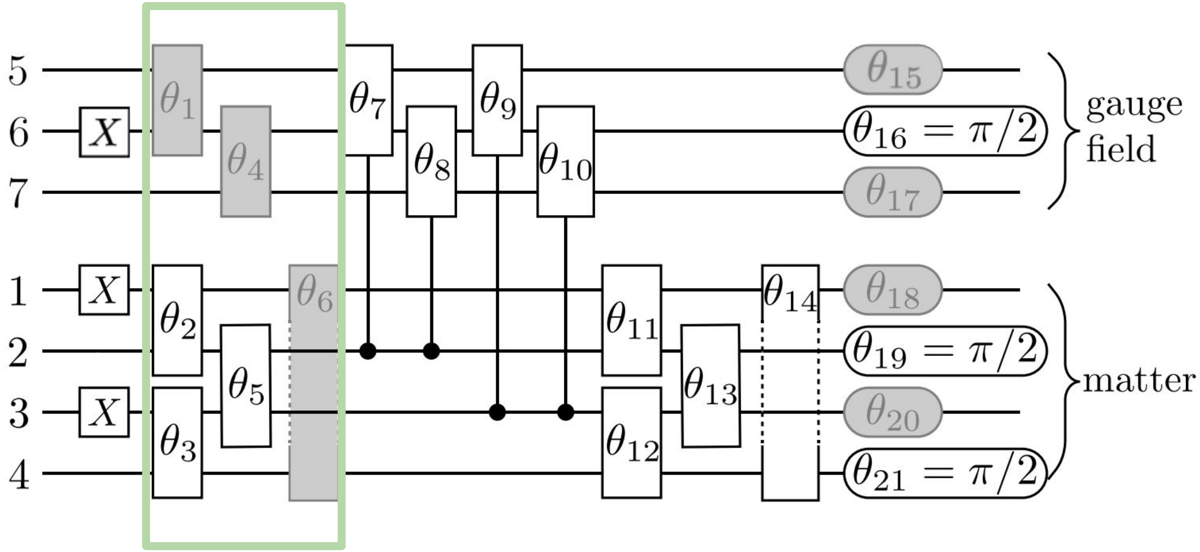
Hamiltonian translated to qua computing language



- initialize vacuum |vvvv>|0>
- bring an excitation into the system

Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

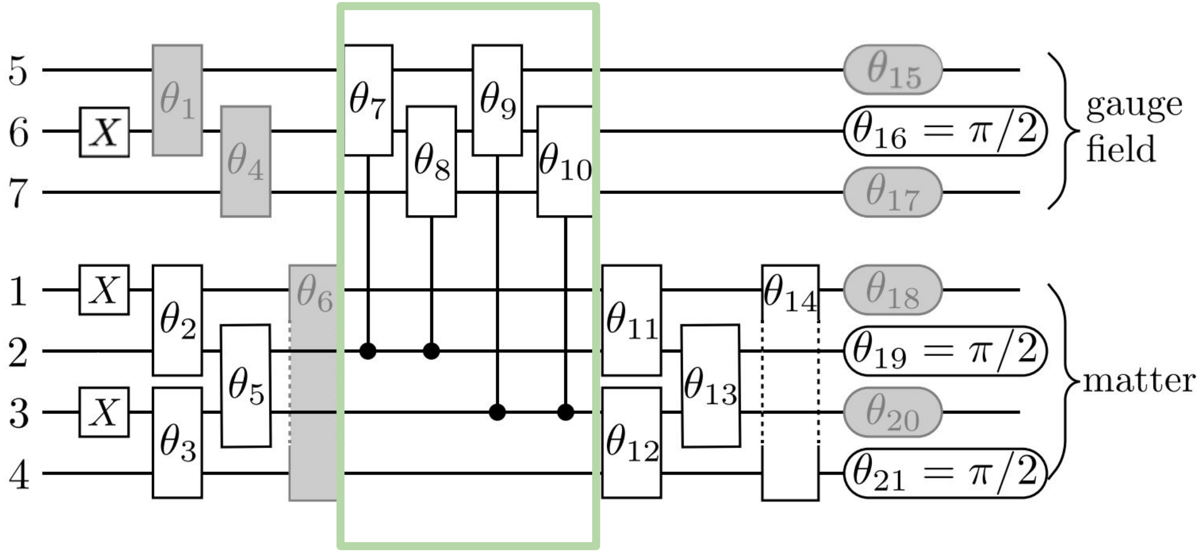
Hamiltonian translated to qua computing language



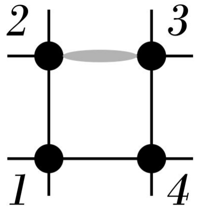
- prepare any possible eigenstate within matter and field subsystems(matter and field) θ_1–θ_6

Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

Hamiltonian translated to qua computing language

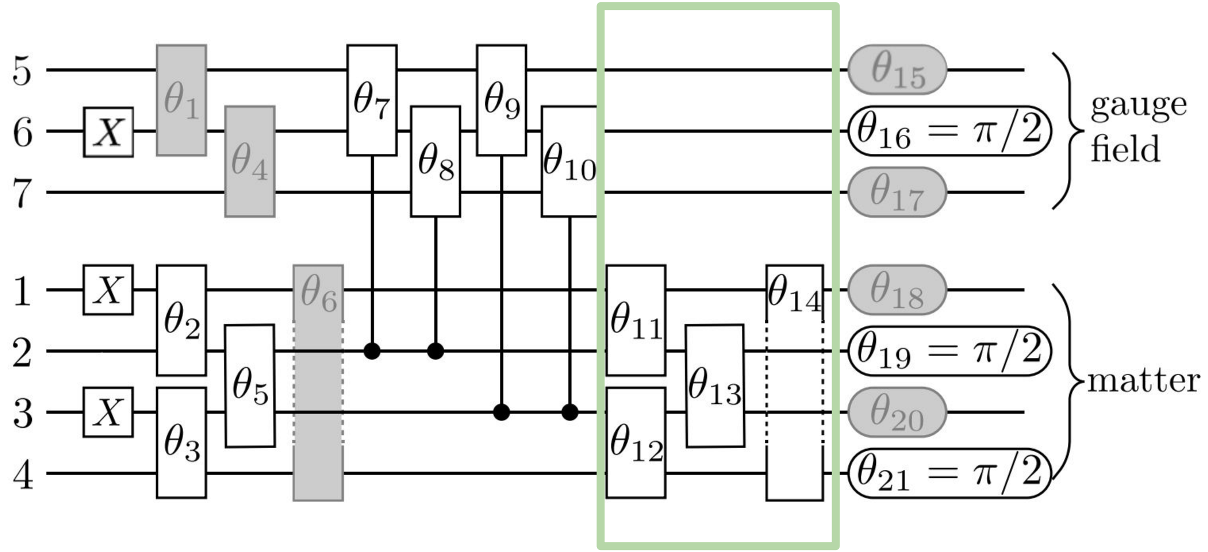


- particle-antiparticle pairs should affect field through controlled iSWAP gates θ_7–θ_{10}



Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

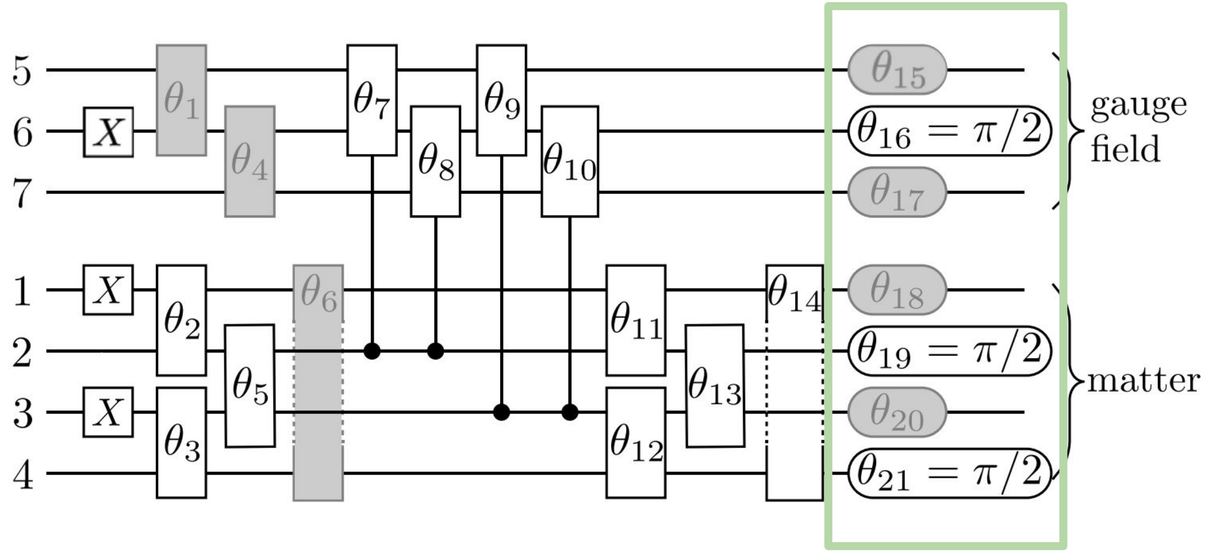
Hamiltonian translated to qua computing language



- final iSWAP gates are applied on the matter qubits again to adjust the state after entangling the two subsystems

Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

Hamiltonian translated to qua computing language



- a layer of single-qubit z rotations is utilized to correct for relative phases.

Paulson, Danny, et al. PRX quantum 2.3 (2021): 030334.

Summary

Summary

- How to create prepare, manipulate and measure a qubit state?
- How to generalize it onto higher Hilbert spaces?
- How to create a non-Abelian gate in non-degenerate system?
- How to use quantum computer for LGT?

Our team!

In these two works participated:

- Joey Lindon
- Logan Cooke
- Mason Protter
- Tian Ooi
- under Lindsay Leblanc supervision

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My physics journey

Hello

My name is **Arina Tashchilina**, and I work with degenerate gases to study fundamental interactions. I am curious to ask and answer questions about gravity, high-energy physics, and artificial gauge fields through tabletop experiments. I am currently working as a Post Doctorate Fellow at the University of Alberta UltraCold Lab with Professor Lindsay Leblanc.

Lab Experience

I am proud to have worked with brilliant research teams under supervision of prof. Lindsay LeBlanc, prof. Barry Sanders, prof. Sergey Moiseev, prof. Aleksey Zheltikov, and prof. Alexander Lvovsky.

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